Technical Note

# Sources of Confusion in the Determination of ASTM Repetitive Member Factors for the Allowable Properties of Wood Products

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**Abstract:** Confusion in the literature about the definition and calculation of repetitive member factors is identified. This confusion casts some doubt on the validity of the 1.15 repetitive member factor permitted in ASTM standards D245 and D1990. **DOI:** 10.1061/(ASCE)ST.1943-541X.0000413. 

**CE Database subject headings:** Wood; Material properties; Reliability.

**Author keywords:** Assembly; Reliability; Redundancy.

## Introduction

It is generally accepted that there should be an upward repetitive member allowable property adjustment. ASTM D245 (2011c) and ASTM D1990 (2011b) specify a 1.15 factor for allowable bending stress. This factor is also listed in ASTM D6555 (2011a, Table 1). In this technical note, sources of confusion regarding appropriate repetitive member factors are identified. This confusion casts some doubt on the validity of the 1.15 value.

The two main sources of confusion discussed are associated with the fact that a full repetitive member factor can be modeled as a combination of subfactors. ASTM D6555 (2011a) identifies three subfactors—load-sharing, composite action, and residual capacity. In the standard, these subfactors are defined, respectively, as "distribution of load among adjacent, parallel members in proportion to relative member stiffness," "interaction of two or more connected wood members that increases the effective section properties over that determined for the individual members," and "ratio of the maximum assembly capacity to the assembly capacity at first failure of an individual member or connection." (Load-sharing and composite action effects continue to exist after first member failure.)

The first confusion is associated with the proper calculation of the load-sharing subfactor. The second confusion is associated with the question of whether the third subfactor, residual capacity, is to be included in the "full" repetitive member factor.

In the next two sections, these two main sources of confusion are described in some detail. In the following section, additional sources of ambiguity are discussed.

Two referees have suggested that in the course of attempting to clear up confusion, the authors might inadvertently be causing additional confusion. To avoid this, the following two statements are made at the outset:

- When it is said that a repetitive member factors might be lower than the 1.15 currently allowed, it is meant that when assembly failure is defined as first member failure, repetitive member factors might be lower than the 1.15 currently allowed.
- When member failure is mentioned, the term refers to the rupture of studs, joists, rafters, and trusses. Thus, for example, even though a T-beam might be composed of a joist, metal connectors, and sheathing, and even though the T-beam might be able to support additional load after the joist ruptures, it is still referred to as first member failure after the joist ruptures. That is, the member in this case is the joist, not the T-beam.

## Confusion about the Method of Calculating a Repetitive Member Adjustment

In ASTM D6555 (2011a, section X2.8), the repetitive member adjustment is determined experimentally by finding an estimate of the fifth percentile of the strength distribution of an assembly (actually a one-sided lower confidence bound on the fifth percentile) in which an assembly's strength is taken to be the load at first member failure, and dividing that by an estimate of the fifth percentile of individual member strength (again, actually a one-sided lower confidence bound on the fifth percentile). In the standard's example, this ratio is 1.11. However, in section 8.4.1 of ASTM D6555, the repetitive member adjustment is defined differently. Rather than the ratio of the fifth percentile of an assembly's strength distribution to the fifth percentile of an individual member's strength distribution, it is defined as the ratio of the fifth percentile of an assembly's strength distribution to the fifth percentile of the minimum of $k$ individual strengths (where an assembly is composed of $k$ members). The fifth percentile of this "minimum of $k$" strength distribution will be lower than the fifth percentile of the strength distribution of a single member, and the calculated repetitive member factor will be larger. Either of the standard's definitions is permissible if subsequently applied correctly in calculating allowable properties, but the ratio described in section 8.4.1 of ASTM D6555 (2011a) cannot be applied to the fifth percentile of an individual member (as is suggested in section 8.5.1 of ASTM D6555). It would have to be applied to the fifth percentile of the distribution

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Note. This manuscript was submitted on June 9, 2010; approved on April 18, 2011; published online on April 20, 2011. Discussion period open until June 1, 2012; separate discussions must be submitted for individual papers. This technical note is part of the ***Journal of Structural Engineering***, Vol. 138, No. 1, January 1, 2012. ©ASCE, ISSN 0733-9445/2012/1-130–133/$25.00.



J. Struct. Eng. 2012.138:130-133.

of the minimum strength of $k$ members. Otherwise, assembly strength would be overestimated.

To make this point clear, note that in the load-sharing simulations described in Verrill and Kretschmann (2010), for strength, stiffness correlations below 1, there is

$$y_{\mathrm{sngl}} > y_{\mathrm{assem}} > y_{\mathrm{mink}} \quad (1)$$

where $y_{\mathrm{sngl}}$ = $q$th percentile of the single member strength distribution, $y_{\mathrm{assem}}$ = $q$th percentile of the load-sharing assembly strength distribution, and $y_{\mathrm{mink}}$ = $q$th percentile of the distribution of the minimum strength of $k$ single members where the assembly is composed of $k$ members and $q = 1, 5, 10$.

To obtain a correct $y_{\mathrm{assem}}$ value, use the following (ratio $R_{\mathrm{ADS}}$ definition):

$$y_{\mathrm{assem}} = (y_{\mathrm{assem}}/y_{\mathrm{sngl}}) \times y_{\mathrm{sngl}} \equiv R_{\mathrm{ADS}} \times y_{\mathrm{sngl}}$$

as is suggested by the example in ASTM D6555 (2011a, section X2.8), or use (ratio $R_{\mathrm{ADM}}$ definition)

$$y_{\mathrm{assem}} = (y_{\mathrm{assem}}/y_{\mathrm{mink}}) \times y_{\mathrm{mink}} \equiv R_{\mathrm{ADM}} \times y_{\mathrm{mink}}$$

but

$$y_{\mathrm{assem}} \neq (y_{\mathrm{assem}}/y_{\mathrm{mink}}) \times y_{\mathrm{sngl}}$$

In fact, because $y_{\mathrm{sngl}} > y_{\mathrm{mink}}$, $(y_{\mathrm{assem}}/y_{\mathrm{mink}}) \times y_{\mathrm{sngl}}$ can be a serious overestimate of $y_{\mathrm{assem}}$. Unfortunately, repetitive member factors reported in the literature are sometimes of the $y_{\mathrm{assem}}/y_{\mathrm{mink}}$ variety and it is common practice to apply these factors to $y_{\mathrm{sngl}}$ values [as suggested in ASTM D6555 (2011a, section 8.5.1)].

In a load-sharing simulation in which assemblies contained five members, Zahn (1970, Table 4) reports load-sharing fractional increases of 0.128, 0.096, 0.125, 0.082, and 0.125. However, these fractional increases are not calculated as

$$(y_{\mathrm{assem}} - y_{\mathrm{sngl}})/y_{\mathrm{sngl}} = R_{\mathrm{ADS}} - 1$$

nor as

$$(y_{\mathrm{assem}} - y_{\mathrm{mink}})/y_{\mathrm{mink}} = R_{\mathrm{ADM}} - 1$$

Instead, they are calculated as

$$(y_{\mathrm{assem}} - y_{\mathrm{mink}})/y_{\mathrm{sngl}}$$

From Zahn (1970, Table 4), the "correct" $R_{\mathrm{ADS}}$ $(y_{\mathrm{assem}}/y_{\mathrm{sngl}})$ load-sharing factors can be calculated: 0.978, 1.077, 0.891, 0.681, and 0.929. Thus, in four of the five cases that Zahn simulated, the assembly was weaker (at the fifth percentile) than an individual member (not accounting for composite action).

Zahn (1970) also references an experiment conducted by Atherton and Corder (1965) at Oregon State. In this experiment, "floors" constructed of 14 beams and 1/2-inch sheathing-grade plywood were loaded to failure. The experimentally determined mean load capacity of five such floors was compared with the mean load capacity of five unsheathed "floors." The load capacity of an unsheathed "floor" was defined to be the minimum of the strengths of the 14 beams that had been assigned to it. Thus, the "increase in load capacity" reported in Zahn (1970, Table 5; taken from the Oregon State report) has an $R_{\mathrm{ADM}}$ nature—that is, it represents the ratio of the strength of a load-sharing assembly to that of a weakest link assembly rather than to the strength of an individual member.

Wolfe and LaBissoniere (1991, Table 3) report load-sharing increases that range from 9 to 47%. However, these are neither $R_{\mathrm{ADS}}$ nor $R_{\mathrm{ADM}}$ values. They constructed three roof assemblies, each containing eight trusses and a gable end. They defined assembly failure as the point at which maximum load was achieved. This point did not correspond to first member failure. (They discuss the failure of trusses within the roof assembly prior to the failure of the assembly.)

The repetitive member factor of Wolfe and LaBissoniere (1991) was the ratio of the assembly load at failure to the smallest maximum load on a member of the assembly that failed in the course of the assembly failure. This is not a ratio of fifth percentile estimates. Also, the denominator is closer to a minimum of $k$ strength value than to an individual member strength value.

Because Wolfe and LaBissoniere (1991) also tested individual trusses to failure, it is possible to calculate $R_{\mathrm{ADS}}$ ratios (although they are at mean population values rather than at fifth percentiles). From Wolfe and LaBissoniere (1991, Table 1), for their three assemblies $R_{\mathrm{ADS}}$ values of 0.84, 1.00, and 1.02 are obtained (0.84 equals the mean of 3,280; 4,210;…; 2,980 divided by the mean of 4,530; 4,360; 4,540; and 4,270). These ratios do not support a repetitive member load-sharing increase (assuming that they begin from a single member allowable property base).

## Confusion about which Literature Values Are Relevant for the ASTM Standards

ASTM D6555 (2011a) notes that repetitive member factors should account for both load-sharing and composite action. However, the failure criterion in ASTM D6555 is currently a serviceability criterion, not a safety criterion. That is, assembly failure is defined to be first member failure, and residual capacity cannot be considered. Unfortunately, the repetitive member factors in the literature that significantly exceed one are, for the most part, based on multiple member failure criteria. In those cases in which the criterion is first member failure, the evidence for a repetitive member factor greater than one is at best mixed.

Verrill and Kretschmann (2010) performed simple load-sharing simulations that defined assembly failure as first member failure, did not include composite action, and assumed perfectly rigid sheathing (so that loads are distributed in proportion to member stiffnesses). They found that under these conditions, $R_{\mathrm{ADS}}$ ratios fall below 1.

Rosowsky and Ellingwood (1992) performed load-sharing simulations that included duration of load effects, but did not include partial composite action. Their simulations yielded repetitive member factors 1.11, 1.33, and 1.53 for three species of wood. However, their definition of assembly failure was the failure of any two members for systems with fewer than eight members, and the failure of two adjacent members for systems with eight or more members. Rosowsky and Ellingwood (1992, Fig. 6), shows that for 10-member assemblies of Douglas fir-larch and southern pine, equal loads lead to higher probabilities of failure for assemblies than for single members, even if assembly failure is defined as the failure of any two members.

Bulleit and Liu (1995) performed simulations that were based on a version of McCutcheon's (1984) beam-spring analog model. Both duration of load and partial composite action were included in their simulations. Bulleit and Liu (1995) defined assembly failure variously as the failure of any one member, of any two members, of any three members, or of any four members. They report three cases (see Bulleit and Liu 1995, Table 9) in which assembly failure was defined to be first member failure. The system factors in these three cases were 1.04, 1.09, and 1.24. Bulleit and Liu (1995, Table 6) also found cases in which system factors were less than 1.



J. Struct. Eng. 2012.138:130-133.

Wheat et al. (1986) created 15 full-scale test floors (three replications of five distinct test floors). For each of these floors, they reported both the ultimate load and the load at first member failure. The mean loads at first member failure can be compared with the population mean strengths of individual members. For the single member means, number 2, 15% moisture content in-grade values were used (Green and Evans 1987). The resulting strength ratios were 0.60, 0.62, 0.79, 0.79, and 0.58 (ratios at the mean, not at the fifth percentile). See Wheat et al. (1986, Tables 1 and 5) for the data needed to calculate these values. Given that Wheat et al. (1986) used number 2 and better boards, these system factors are probably overestimates.

In a thorough and insightful analytical study, Rosowsky and Yu (2004) made use of an extension (Yu 2003) of McCutcheon's (1984) beam-spring analog model to evaluate a portfolio of wall systems. Rosowsky and Yu (2004) considered the following four ratios related to Verrill and Kretschmann's (2010) $R_{\mathrm{ADS}} = y_{\mathrm{assem}}/y_{\mathrm{sngl}}$ and $R_{\mathrm{ADM}} = y_{\mathrm{assem}}/y_{\mathrm{mink}}$:

$$K_{\mathrm{PY}} \approx (5\text{th percentile of ultimate system strength})/y_{\mathrm{assem}} \quad (2)$$

$$K_{\mathrm{LS}} \approx y_{\mathrm{assem}}/y_{\mathrm{mink}} = R_{\mathrm{ADM}} \quad (3)$$

$$K_{\mathrm{NMEM}} \approx y_{\mathrm{mink}}/y_{\mathrm{sngl}} \quad (4)$$

$$K_{\mathrm{PCA}} \equiv (5\text{th percentile of T–beam strength})/y_{\mathrm{sngl}} \quad (5)$$

where PY stands for post yield; LS stands for load-sharing; NMEM stands for number of members; and PCA stands for partial composite action. Eqs. (2)–(4) are only approximations because the $y$s in Verrill and Kretschmann (2010) do not include partial composite action.

Rosowsky and Yu (2004) calculated their raw repetitive member factor as

$$K_{\mathrm{PY}} \times K_{\mathrm{LS}} \times K_{\mathrm{NMEM}} \times K_{\mathrm{PCA}}$$

They found that, by far, the largest contributor to a repetitive member factor greater than one was $K_{\mathrm{PY}}$. That is, residual capacity is the greatest contributor. However, D6555 (ASTM 2011a) does not currently permit one to include residual capacity in the calculation of a repetitive member factor. When $K_{\mathrm{PY}}$ is set to one as it must be if assembly failure is defined as first member failure, then the work by Rosowsky and Yu (2004) yields repetitive member factors (that include partial composite action) that fall below 1 (see Rosowsky and Yu 2004, Table 3; Rosowskyet al. 2005). This is driven by the small value of $K_{\mathrm{NMEM}}$, essentially the ratio of the fifth percentile of the minimum of the strengths of $k$ members to the fifth percentile of the strength distribution of a single member (where $k$ members are in the assembly).

$R_{\mathrm{ADS}} = y_{\mathrm{assem}}/y_{\mathrm{sngl}}$ in Verrill and Kretschmann (2010) is approximately equal to $K_{\mathrm{LS}} \times K_{\mathrm{NMEM}}$ in Rosowsky and Yu (2004). Both Verrill and Kretschmann and Rosowsky and Yu found that this product decreases with increasing modulus of rupture coefficient of variation (COV) (see Rosowsky and Yu 2004, Fig. 8).

There are studies that calculate the repetitive member factor as done in section X2.8 of ASTM D6555 (fifth percentile tolerance limit for the assembly strength distribution to the fifth percentile tolerance limit of an individual member strength distribution) and obtain values greater than 1.0 (e.g., Bohnhoff et al. 1991, Table 4). Ratios of fifth percentile estimates can differ significantly from ratios of tolerance limits. Compare the 1.27 and 1.40 values in the last column of Bohnhoff et al. (1991, Table 4). Furthermore, although their ratio of fifth percentile estimates is greater than one, their ratio of means is less than 1.

Using finite-element methods, Folz and Foschi (1989) obtained an average system factor of 1.38 for systems in which assembly failure was first member failure. However, they also obtained system factors that lay near or considerably below 1 in specific cases (see Folz and Foschi 1989, Table 7).

Thus, it is not claimed that no cases exist in which the repetitive member factor should be greater than 1.0. It is claimed, however, that confusion exists in the literature regarding the definition of a repetitive member factor, and that it is by no means clear (at least if calculations are restricted to adjustments due to load-sharing and partial composite action, and a definition of assembly failure as first member failure) that the factor should, in general, be greater than 1.0 when it is defined as $y_{\mathrm{assem}}/y_{\mathrm{sngl}}$, the ratio of the fifth percentile of the assembly strength distribution to the fifth percentile of the individual member distribution (as it is commonly applied but not always calculated).

## Additional Sources of Ambiguity

Even if an $R_{\mathrm{ADS}}$ (rather than an $R_{\mathrm{ADM}}$) definition of the repetitive member factor is settled on, and it is agreed for the current purposes of ASTM D6555 (2011a), that only load-sharing and composite action are included in this factor, a wide range of $R_{\mathrm{ADS}}$ values can be obtained. This point has been made previously by (at the least) Folz and Foschi (1989), Bulleit and Liu (1995), Rosowsky and Yu (2004), Verrill and Kretschmann (2010), and comments in the standard itself. For example:

- As the number of members in an assembly increases, the repetitive member factor decreases. This is in accord with intuition and has been observed by Folz and Foschi (1989), Rosowsky and Ellingwood (1991), Bulleit and Liu (1995), Rosowsky and Yu (2004), and Verrill and Kretschmann (2010).
- As expected the repetitive member factor increases as the correlation between strength and stiffness increases—as load-sharing becomes more effective. This behavior has been observed by Folz and Foschi (1989), Bulleit and Liu (1995), and Verrill and Kretschmann (2010).
- As the modulus of rupture (MOR) COV increases, $R_{\mathrm{ADS}}$ values decrease. In fact, for large COVs, load-sharing assembly strengths can be much less than single member strengths. It is true that $R_{\mathrm{ADM}}$ values do increase with COV. That is, the advantage of load-sharing assemblies over weakest link assemblies (but not over single members) increases with increasing COV. Intuitions about the effect of an increase in the coefficient of variance of modulus of rupture have been, in some cases, incorrect (see Verrill and Kretschmann 2009). For a one member failure criterion, a decline in the repetitive member factor with an increase in MOR COV has been observed by Folz and Foschi (1989), Bulleit and Liu (1995), Rosowsky and Yu (2004), and Verrill and Kretschmann (2010). For a two adjacent members failure criterion, a decline in the repetitive member factor with an increase in MOR COV has been observed by Rosowsky and Ellingwood (1991).

  In contrast, for various failure criteria that require more than one member failure, Bulleit and Liu (1995) report that the repetitive member factor increases as modulus of rupture COV increases.
- Folz and Foschi (1989) and Bulleit and Liu (1995) have identified a number of other variables (for example, sheathing



J. Struct. Eng. 2012.138:130-133.

thickness and fastener stiffness) that can have a significant effect on the repetitive member factor.

## Summary and Conclusions

Confusion in the literature about the proper choices for ASTM repetitive member factors has been identified. It appears that at least some of the support for an 1.15 ASTM repetitive member factor may stem from this confusion. First, engineers might have $R_{ADM}$ values in mind rather than the appropriate $R_{ADS}$ values. Second, they might have $R_{ADS}$ values in mind that are appropriate for a multiple member failure criterion but not for a first member failure criterion.

This work suggests that authors must be quite careful when they report the results of repetitive member studies, and standards bodies must be even more careful when they interpret these results. For example, answers to the following questions must be clear:

1. Which of the subfactors—load-sharing, partial composite action, residual capacity—does a reported system factor include?
2. What is the definition of assembly failure? Is it first member failure as specified in ASTM D6555 (2011a), or the failure of two members, or the failure of two adjacent members, or maximum assembly load capacity, or…?
3. What definition of repetitive member factor is used? If the repetitive factor is calculated as a ratio of strengths, then
   - Is it calculated as $R_{ADS} = y_{assem}/y_{sngl}$ or $R_{ADM} = y_{assem}/y_{mink}$?
   - If the $R_{ADM}$ definition is used, how is the factor to be applied?
   - At what percentile (e.g., 50th, 5th,…) is the ratio calculated? ASTM D6555 (2011a) specifies the fifth percentile, but some data sets only permit it to be calculated at the mean.
   - Is it a ratio of fifth percentiles or a ratio of tolerance bounds? ASTM D6555 (2011a) specifies the fifth percentile in section 8.4.1.2 but uses tolerance bounds in the example in section X2.8.
4. How many members are in the assembly?
5. What distributions of single member strength are considered?
6. What is the COV of the individual member strength distributions?
7. What is the correlation between strength and stiffness?

The correct repetitive member factor can depend heavily upon the answers to these questions.

Taken together, these dependencies suggest that a single repetitive member factor is unlikely to be appropriate. This point has been made previously by Folz and Foschi (1989), Bulleit and Liu (1995), Rosowsky and Yu (2004), Verrill and Kretschmann (2010), and in ASTM D6555 (2011a).

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J. Struct. Eng. 2012.138:130-133.