United States
Department of
Agriculture

Forest Service

Forest
Products
Laboratory

Research
Paper
FPL-RP-616

Estimating the Board Foot to Cubic Foot Ratio

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Abstract

Certain issues in recent softwood lumber trade negotiations have centered on the method for converting estimates of timber volumes reported in cubic meters to board feet. Such conversions depend on many factors; three of the most important of these are log length, diameter, and taper. Average log diameters vary by region and have declined in the western United States due to the growing scarcity of large diameter, old-growth trees. Such a systematic reduction in size in the log population affects volume conversions from cubic units to board feet, which makes traditional rule of thumb conversion factors antiquated. In this paper we present an improved empirical method for performing cubic volume to board foot conversions.

Keywords: Scribner scaling, diameter, length, taper, truncated cone, smoothing, calibration

Contents

March 2004

Verrill, Steve P.; Herian, Victoria L.; Spelter, Henry N. 2004. Estimating the board foot to cubic foot ratio. Res. Pap. FPL-RP-616. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 18 p.

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SI conversion factors

Inch–pound	Conversion factor	SI unit
inch	25.4	millimeter
foot	0.3048	meter
cubic foot	0.0283	cubic meter

Estimating the Board Foot to Cubic Foot Ratio

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1 INTRODUCTION

In the United States, output-based log measurement systems for estimating timber volumes are still in common use. The resulting estimates are expressed in board feet. In other countries, full-log volume systems (firmwood scales) are used, and the results are reported in cubic meters. These approaches to log measurement are fundamentally different. Output-based scales estimate only the portion of a log recoverable as finished primary product (i.e., lumber) based on the small-end diameter. Cubic rules measure the full volume of sound wood, inclusive of lumber, chips, and sawdust. This leads to differences between the approaches that can be summed up as follows:

1. In an output-based log scale, a log's volume is approximated as a cylinder based on the small-end diameter. In firmwood scales, the full volume is approximated via methods that depend on both end diameters. Thus, firmwood scales take into account the volume contained in the tapered portion outside of the central cylinder, while output-based scales generally neglect this material.

2. Because of differences in measurement protocols (e.g., measurements are rounded down in the Scribner scaling rules applied on the U.S. West Coast), the reported length and diameter of a log can be smaller in some output-based scales than in a firmwood-based scale.

3. Output-based measures estimate the board feet of lumber that can be extracted from a cylinder based on cutting diagrams that exclude a portion of the cylinder as waste. In firmwood scales the entire volume is taken into account.

4. Because of the different philosophies underlying output scales (based on lumber yield) and firmwood scales (based on fiber yield), trim and defect deductions are often greater in output-based scales than in firmwood-based scales.

Differences among scaling systems and the shortcomings of output-based scales in particular have long been noted (Rapraeger, 1950). The greater stability and consistency of measurements obtained by firmwood-based cubic approaches have also been noted, leading to recommendations to abandon output-based scaling in favor of cubic scaling (Orchard, 1953). Such transitions have occurred in Canada and partially in the United States where Federal land management agencies have switched operations to cubic scaling (USDA Forest Service, 1991). Nevertheless, output-based board foot rules are still widely used in the United States, and this poses problems when volumes and values calculated under the two systems need to be compared.

Past approaches to volume conversions of log populations implicitly rested on the assumption that the underlying size characteristics of the timber supply were stable. Thus, conversions could be calculated by the application of constants derived from historical observations of the resource.

A more flexible approach to conversions between scales was undertaken by Cahill (1984), who developed empirical equations relating volume in cubic feet to volume in board feet as a function of scaling diameter. While an improvement over the use of historical rules of thumb, this approach did not explicitly incorporate the effects of other variables such as length and taper. In our current work we propose a model that does take into account log length and taper. We account for other variables (e.g., defect and trim allowances) by calibrating the model against a large West Coast population of logs for which both Scribner output-based rule measurements (Decimal C version) and metric cubic measurements are available. We have produced a web-based computer program that implements this calibrated model.

Our work suggests that the model would be useful to economists who wish to convert values of log populations denominated in Scribner board feet in the coastal western United States to/from values expressed in British Columbia cubic units. However, for other regions where the short log version of the Scribner formula is applied, the model needs to be calibrated against additional data sets. We hope to accomplish this calibration in the near future.

2 THE $F_3 \times F_2 \times F_1$ MODEL

We would like to develop a model that predicts the board foot to cubic foot ratio for a population of logs from the population average* small-end diameter, length, and taper. To do so we first concentrate on a single log and then generalize the approach to a population of logs. For a single log, ignoring trim and defects, we have

$$\frac{\text{board feet from table}}{\text{cubic feet}} \approx F_3 \times F_2 \times F_1 \tag{1}$$

where

$$\begin{aligned} F_1 &\equiv \text{(cylinder volume)/(truncated cone volume)} \\ F_2 &\equiv \text{(approximate cylinder volume)/(cylinder volume)} \\ F_3 &\equiv \text{(board feet from table)/(approximate cylinder volume)} \end{aligned}$$

These definitions are depicted graphically in Figure 1. The "truncated cone volume" is an approximation to the full volume of the log.† If the truncated cone has small-end diameter D_S and length L, then the "cylinder volume" is the volume of a cylinder with diameter D_S and length L. In West Coast Scribner rules, the diameter and length are truncated down so the "approximate cylinder volume" will be less than the cylinder volume. In East Coast rules, the diameter is rounded to the nearest inch and the length is rounded to the nearest foot. In this case the approximate cylinder volume can be either above or below the cylinder volume. The "board feet from the table" is obtained by entering the Scribner table via the diameter and length of the approximate cylinder.

3 THE F_1 FACTOR

In the Appendix we show that

$$F_1 = 1/(1 + R + R^2/3) \tag{2}$$

where $R = (T \times L)/D_S$, T = taper, L = length, and D_S = small-end diameter.

*These averages should be weighted averages where the weight associated with a log is proportional to the log's cubic volume.

†We realize that this is only an approximation. An alternate approximation would be the volume of a frustrum of a paraboloid. This leads to the Smalian formula. In general, the two approaches yield very similar results.

4 $F_3 \times F_2$

We have

$$\begin{aligned} F_3 \times F_2 &= \frac{\text{board feet from table}}{\text{cylinder volume}} \\ &= \frac{\text{board feet from table}}{\pi D_S^2 L/4} \end{aligned}$$

This ratio can be calculated exactly from the Scribner table and the relevant rounding rules. For example, consider a 21.3-foot log with a 7.6-inch small-end diameter.

Under West Coast rounding rules, the length and diameter are truncated to 21 feet and 7 inches, and the $F_3 \times F_2$ value[‡] can be calculated as

$$F_3 \times F_2 = 30/(\pi \times (7.6/12)^2 \times 21.3/4) = 4.47$$

where the 30 comes from the 21-foot length, 7-inch diameter entry in the Scribner table.

Under East Coast rounding rules, the length is rounded to the nearest foot – 21 feet, and the diameter is rounded to the nearest inch – 8 inches. In this case we have

$$F_3 \times F_2 = 40/(\pi \times (7.6/12)^2 \times 21.3/4) = 5.96$$

where the 40 comes from the 21-foot length, 8-inch diameter entry in the Scribner table.

5 APPLYING THE $F_3 \times F_2 \times F_1$ MODEL TO A POPULATION OF WEST COAST LOGS

The Scribner table is not a smooth function of log length and diameter. It includes values for 1-foot increments in log length and 1-inch increments in small-end diameter. Board foot values remain the same for increasing lengths and diameters and then suddenly jump.[§] These jumps in the Scribner table board foot values lead to jagged plots of $F_3 \times F_2$ values — while Scribner board foot values are staying constant, the cylinder volumes are increasing so the $F_3 \times F_2$ values are declining; then when the Scribner value suddenly jumps, the $F_3 \times F_2$ value suddenly jumps as well. A plot of the resultant unsmooth $F_3 \times F_2$ surface given West Coast diameter and length truncation rules is presented in Figure 4.

One approach to applying the $F_3 \times F_2 \times F_1$ model to a population of logs would be to calculate the volume weighted average small-end diameter, length, and taper for the population. These values would then be plugged into Equation (2) to obtain an F_1 value. The Scribner table and the volume weighted average small-end diameter and length would be used to obtain an $F_3 \times F_2$ value. Then the estimated board foot to cubic foot ratio for the population would be reported as $F_3 \times F_2 \times F_1$. A problem with this approach is the unsmooth nature of the $F_3 \times F_2$ surface. Because of this roughness, very small changes in the population means could lead to large swings in the conversion factor. For example, as noted in footnote 4, if a population had a volume weighted mean length of

[‡]Note that since $F_3 \times F_2$ equals the board feet that can be obtained from a cylinder divided by the volume of the cylinder, and board feet is reported in feet × feet × inches rather than cubic feet, under West Coast rules the value of $F_3 \times F_2$ is bounded above by 12.

[§]For example, the table entry for a log of length 12 feet (see Figure 2) and small-end diameter 9 inches is 30. For length 12 feet and diameter 10 or 11 inches, the entries are 40; for length 12 feet and diameter 12 inches, the entry is 60. For logs of diameter 9 inches (see Figure 3) and lengths 16 through 18 feet, the entries are 40; for logs of diameter 9 inches and lengths 19 through 22 feet, the entries are 50; and so on.

12 feet and volume weighted mean small-end diameter of 11.99 inches, the calculated $F_3 \times F_2$ would be about two-thirds (40/60) of the $F_3 \times F_2$ value for a population with volume weighted mean length of 12 feet and volume weighted mean small-end diameter of 12 inches. This is unacceptable. To avoid this problem we smooth the $F_3 \times F_2$ surface.

6 SMOOTHING THE $F_3 \times F_2$ SURFACE

Many techniques are available for smoothing a two-dimensional surface. One technique in common use is the Gaussian kernel smoothing approach. In essence this method replaces the value $f(x_0, y_0)$ in a surface with

$$\mathrm{smf}(x_0, y_0) = \frac{\int_a^b \int_a^b f(x,y) \times \frac{1}{2\pi} \exp(-(x-x_0)^2/\sigma_x^2) \exp(-(y-y_0)^2/\sigma_y^2) dx\, dy}{\int_a^b \int_a^b \frac{1}{2\pi} \exp(-(x-x_0)^2/\sigma_x^2) \exp(-(y-y_0)^2/\sigma_y^2) dx\, dy} \tag{3}$$

In words, this approach replaces the value at length x_0 and diameter y_0 in the $F_3 \times F_2$ surface with a weighted average[¶] of the nearby $F_3 \times F_2$ values where the weights decline as the lengths and diameters move away from x_0, y_0. The parameters σ_x and σ_y determine how rapidly the weights decline. Small σ values lead to weights that decline rapidly, the smoothing is minimal, and the smoothed surface remains relatively jagged. Large σ values lead to weights that decline slowly and the surface is highly smoothed. In the limit, as the σ's get very large, the smoothed surface becomes a horizontal plane.

7 OPTIMAL SMOOTHING PARAMETER

In our work the parameter α (recall from footnote 5 that $\sigma_x = \alpha \times x_0$, $\sigma_y = \alpha \times y_0$) determines the smoothness of the smoothed $F_3 \times F_2$ surface. Figures 5 and 6 present the smoothed surfaces that result for $\alpha = 0.05$ and $\alpha = 0.10$. As α increases, the surface becomes smoother. How do we choose the appropriate level of smoothness? When $\alpha = 0$, we get the original jagged surface, but when α gets very large, we get the unweighted average of all of the $F_3 \times F_2$ values, which is a poor representation of what is going on. Obviously we need α to take on some intermediate value.

The optimal α depends upon what we want to do, and it will be objective and data dependent. In our case we were working with a data set of 455,382 West Coast logs. We wanted our model to do a good job of predicting the board foot to cubic foot ratios for subpopulations of these logs. We proceeded as follows. First we sorted the logs by small-end diameter. Then we formed 45 subpopulations of 10,000 logs each. The first subpopulation contained the logs with the 10,000 largest small-end diameters. The next subpopulation contained the 10,000 logs with the next largest small-end diameters and so on. For each subpopulation we calculated volume weighted average small-end diameter, length, and taper. We used these averages in Equation (2) to obtain an F_1 value. We used the average length as x_0 and the average small-end diameter as y_0 to enter the smoothed Scribner table (smoothed using the chosen value for α) and obtained an $F_3 \times F_2$ value. We calculated the actual gross board feet for the subpopulation by adding the gross board foot values for all the logs in the subpopulation. We calculated the actual net cubic feet for the

[¶]In our implementation we did not actually perform the integration in Equation (3). This would be a difficult task for a numerical integration routine because of the jagged nature of the surface. Instead we performed a Monte Carlo integration. We drew 10000 (x, y) values from a bivariate normal distribution with mean (x_0, y_0), $\sigma_x = \alpha \times x_0$, $\sigma_y = \alpha \times y_0$, and covariance$(x, y) = 0$, where α is the "smoothing parameter" (see Section 7). We then calculated $F_3 \times F_2$ at each of the sampled (x, y) pairs that lay in the table and averaged the results.

subpopulation by adding the net cubic foot values for all the logs in the subpopulation. The actual gross board foot to net cubic foot ratio for the subpopulation was the ratio of these two totals. The predicted ratio was $F_3 \times F_2 \times F_1$. We then performed a regression that fit the 45 actual ratios to the 45 predicted ratios. The root mean squared error (RMSE) from this regression was the measure of the performance of the smoothing parameter. The smaller the RMSE, the better. Figure 7 is a plot of RMSE versus α value. For this data set and the 45 subpopulations, the optimal smoothing parameter value was approximately 0.025.

We repeated this procedure for subpopulations of size 20,000, 40,000, and 90,000. The characteristics of the resulting subpopulations are displayed in Tables 1 to 4. The corresponding RMSE versus smoothing parameter plots suggested that a smoothing parameter of 0.08, while not optimal in all cases, would be a reasonable compromise and would lead to good RMSE values in all cases considered.

8 CALIBRATION

In our data set we had information that permitted us to calculate the net board feet, the gross board feet, and the net cubic feet for each log. In the top half of Figure 8, we plot the observed gross board foot to net cubic foot ratio (BFDCFGN) versus $F_3 \times F_2 \times F_1$ for 1,000 logs randomly selected from the population of 455,382 logs. The solid line in the plot is the $y = x$ line.

In the bottom half of Figure 8, we plot the observed net board foot to net cubic foot ratio (BFDCFNN) versus $F_3 \times F_2 \times F_1$ for the same 1,000 randomly selected logs. Again, the solid line in the plot is the $y = x$ line.

To calibrate the model to the data we performed regressions of BFDCFGN and BFDCFNN on $F_3 \times F_2 \times F_1$ for 20,000 logs randomly selected from the full population.

The resulting calibration equations are

$$\text{BFDCFGN} = -0.01048 + 0.9742 \times F_3 \times F_2 \times F_1 \tag{4}$$

and

$$\text{BFDCFNN} = 0.1316 + 0.9255 \times F_3 \times F_2 \times F_1 \tag{5}$$

These models were then applied[||] to predicting the ratios associated with the 45 subpopulations of size 10,000, 22 subpopulations of size 20,000, 11 subpopulations of size 40,000, and 5 subpopulations of size 90,000 described above. We provide the resulting plots of predicted versus observed ratios in Figures 9 to 12. The corresponding root mean squared errors (RMSEs) for the BFDCFGN plots are 0.129, 0.118, 0.090, and 0.073. Those for the BFDCFNN plots are 0.132, 0.121, 0.101, and 0.085. For the full data set the observed BFDCFGN is 4.97 and the predicted value is 5.02. The observed BFDCFNN is 4.85 and the predicted is 4.91.

When we calibrate Cahill's model to the data set of 20,000 randomly selected logs and then apply the model to the subpopulations, the BFDCFGN RMSE values are 0.249, 0.185, 0.173, and 0.082. The BFDCFNN RMSE values are 0.257, 0.185, 0.171, and 0.081. The predicted BFDCFGN value for the whole population is 4.66[**] (4.97 observed), and the predicted BFDCFNN is 4.58[**] (4.85 observed).

[||] To use Equations (4) and (5), we calculate F_1 via Equation (2) and net cubic volume weighted average diameter, length, and taper. $F_3 \times F_2$ is taken to be the value of the smoothed $F_3 \times F_2$ surface (with the smoothing parameter set equal to 0.08) at the net cubic volume weighted average diameter and length.

[**] As suggested by Cahill, we calculated these values using the quadratic mean diameter. If we instead use the cubic volume weighted average diameter, we obtain 5.10 rather than 4.66, and 4.99 rather than 4.58.

9 PROBLEMS

The most obvious problem with models (4) and (5) is that the calibration coefficients were determined for a particular data set. *A priori* we have no way of knowing whether these values are appropriate for other West Coast data sets. We would expect them to be even less appropriate for East Coast data sets that involve different rounding rules. This question can only be resolved by looking at other data sets.

Two problems are associated with our use of weighted averages. First there is a theoretical problem. For the ith log in a population of logs, we have the mathematical equation

$$\mathrm{BFDCFGN}_i = f(\mathrm{diameter}_i, \mathrm{length}_i, \mathrm{taper}_i) + \epsilon_i \tag{6}$$

where $\mathrm{BFDCFGN}_i$ is the gross board foot to net cubic foot ratio for the log. The function f incorporates the relation between a truncated cone and a cylinder, the rounding of diameter and length values, and the smoothed Scribner table. The error, ϵ_i, incorporates the departure of the log shape from that of a truncated cone, measurement error, and so on. Although we might expect this relationship to hold for individual logs, it is not immediately clear that it will yield good results for a population of logs. In particular, *a priori* we do not know whether Equation (6) implies

$$\mathrm{BFDCFGN}_{\mathrm{pop}} \approx f(\bar{D}, \bar{L}, \bar{T}) \tag{7}$$

where

$$\bar{D} = \sum_{i=1}^{n} \mathrm{diameter}_i \times W_i = \text{the volume weighted average diameter}$$

(the weight W_i equals (cubic feet)$_i$/(total cubic feet))

$$\bar{L} = \sum_{i=1}^{n} \mathrm{length}_i \times W_i = \text{the volume weighted average length}$$

$$\bar{T} = \sum_{i=1}^{n} \mathrm{taper}_i \times W_i = \text{the volume weighted average taper}$$

However, we do have

$$\mathrm{BFDCFGN}_{\mathrm{pop}} = \sum_{i=1}^{n} (f(\mathrm{diameter}_i, \mathrm{length}_i, \mathrm{taper}_i) + \epsilon_i) \times W_i \tag{8}$$

or, after a Taylor series expansion,

$$\begin{aligned} \mathrm{BFDCFGN}_{\mathrm{pop}} &= \sum_{i=1}^{n} [f(\bar{D}, \bar{L}, \bar{T}) + (\partial f/\partial D)(\mathrm{diameter}_i - \bar{D}) \\ &+ (\partial f/\partial L)(\mathrm{length}_i - \bar{L}) + (\partial f/\partial T)(\mathrm{taper}_i - \bar{T})] \times W_i \\ &+ \text{ weighted average of second order Taylor series terms} + \sum_{i=1}^{n} \epsilon_i W_i \end{aligned} \tag{9}$$

Thus,

$$\mathrm{BFDCFGN}_{\mathrm{pop}} = f(\bar{D}, \bar{L}, \bar{T}) \tag{10}$$

$$+ \text{ weighted average of second order terms} + \sum_{i=1}^{n} \epsilon_i W_i$$

and approximation (7) does hold provided that the weighted average of second order Taylor series terms and the weighted average of the errors are small relative to $f(\bar{D}, \bar{L}, \bar{T})$. Now, *a priori*, we do not know whether this proviso holds. However, we have found *empirically* that result (7) is indeed a good approximation for our population of 455,000 logs and the subpopulations that we considered (see Figures 9 to 12). Of course we will have to test whether this approximation works well for other populations before we recommend the procedure in general.

The second problem associated with weighted averages is a practical one. How does one obtain estimates of these weighted averages of diameter, length, and taper for a population? Further, if one has sufficient information to obtain such averages, isn't one likely to have enough information to calculate the board foot to cubic foot ratio almost exactly? After all, to calculate the averages exactly one would have to have diameter, length, and taper for every log in the population.

Our answer is that it is uncommon to have all of these data for a region. More often, for economic analyses, these values have to be assumed or extracted from small samples. Equations (4) and (5) offer a way to investigate the effects of errors in the assumptions, or biases in the limited data.

10 COMPUTER PROGRAM

We have developed a FORTRAN computer program that implements our estimation procedures. Given cubic volume weighted average diameter, length, and taper values (before truncations and before defect deductions), the program uses Equation (2) to calculate F_1. It smooths the $F_3 \times F_2$ surface at the volume weighted average diameter and length to obtain an $F_3 \times F_2$ value, and it then reports the predicted BFDCFGN ratio as BFDCFGN $= -0.01048 + 0.9742 \times F_3 \times F_2 \times F_1$ and the predicted BFDCFNN ratio as BFDCFNN $= 0.1316 + 0.9255 \times F_3 \times F_2 \times F_1$.

We have put a Web forms front end on this program and it can be run at `http://www1.fpl.fs.fed.us/conversion.html`. (If your domain name server fails to resolve this address, you should try `http://128.104.77.229/conversion.html`.)

11 CONCLUDING REMARKS

We have developed a method for estimating the board foot to cubic foot ratio from the volume weighted average diameter, length, and taper of a log population. Currently we know that this method performs well for a particular West Coast population of logs. We will be attempting to extend it to a wider population of West Coast logs and to modify it to be appropriate for East Coast log populations. We have implemented the current method as a FORTRAN computer program that can be run over the Web.

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APPENDIX — THE F_1 FACTOR

Here we justify the approximation

$$F_1 \approx 1/(1 + R + R^2/3)$$

where $R = (T \times L)/D_S$, T is the taper of the log, L is the length of the log, D_S is the small-end diameter of the log, and F_1 is the ratio of the volume of the central cylinder of the log to the full volume of the log.

To establish this we first note that

$$\text{truncated cone volume} = \pi L(D_S^2 + D_S D_l + D_l^2)/12 \qquad (11)$$

where D_S is the small-end diameter and D_l is the large-end diameter. Next we have

$$D_l = D_S + T \times L$$

Replacing D_l in Equation (11) by $D_S + T \times L$ we obtain

$$\begin{aligned}\text{truncated cone volume} &= \pi L(D_S^2 + D_S(D_S + TL) + (D_S + TL)^2)/12 \\ &= \pi L(3D_S^2 + 3D_S TL + (TL)^2)/12\end{aligned}$$

Thus,

$$\begin{aligned}F_1 &\approx \text{the ratio of the volume of the central cylinder of a truncated cone} \\ &\quad\ \text{to the full volume of the truncated cone} \\ &= (\pi L D_S^2/4)/(\pi L(3D_S^2 + 3D_S TL + (TL)^2)/12) \\ &= 1/(1 + TL/D_S + (TL/D_S)^2/3) = 1/(1 + R + R^2/3)\end{aligned}$$

where $R = TL/D_S$, which is what we wanted to establish.

Note that the adequacy of this approximation will depend on the extent to which the shape of the log can be approximated by a truncated cone.

Table 1. Forty-five subpopulations of size 10,000 ordered by increasing metric log diameter

Metric volume weighted average			Scribner defect	Total gross Scribner	Total net Scribner	Total net metric	BFDCF	BFDCF
diameter[1]	length[2]	taper[3]	fraction[4]	volume[5]	volume[6]	volume[7]	GN[8]	NN[9]
6.30	38.35	0.0862	0.697E-2	0.583E+6	0.579E+6	0.137E+6	4.26	4.23
6.30	35.14	0.1022	0.953E-2	0.486E+6	0.481E+6	0.123E+6	3.95	3.91
6.30	30.63	0.1335	0.278E-1	0.374E+6	0.364E+6	0.111E+6	3.36	3.27
6.84	36.93	0.0995	0.836E-2	0.560E+6	0.555E+6	0.150E+6	3.73	3.70
7.09	39.25	0.0793	0.167E-2	0.654E+6	0.653E+6	0.164E+6	3.98	3.97
7.09	32.97	0.1228	0.130E-1	0.484E+6	0.478E+6	0.141E+6	3.42	3.38
7.65	37.54	0.0968	0.782E-2	0.652E+6	0.647E+6	0.181E+6	3.60	3.57
7.87	39.42	0.0784	0.633E-2	0.771E+6	0.766E+6	0.194E+6	3.97	3.94
7.96	33.45	0.1303	0.208E-1	0.572E+6	0.560E+6	0.177E+6	3.23	3.17
8.66	37.55	0.0826	0.898E-2	0.791E+6	0.784E+6	0.214E+6	3.69	3.66
8.66	37.86	0.0840	0.119E-1	0.809E+6	0.799E+6	0.215E+6	3.76	3.71
9.04	35.69	0.1109	0.213E-1	0.803E+6	0.785E+6	0.227E+6	3.54	3.46
9.45	36.73	0.0824	0.574E-2	0.967E+6	0.961E+6	0.240E+6	4.03	4.00
9.45	38.56	0.0813	0.106E-1	0.107E+7	0.106E+7	0.258E+6	4.14	4.09
9.45	38.24	0.0810	0.435E-2	0.104E+7	0.103E+7	0.255E+6	4.07	4.05
9.48	34.89	0.1200	0.279E-1	0.889E+6	0.864E+6	0.241E+6	3.68	3.58
10.24	37.38	0.0864	0.109E-1	0.131E+7	0.129E+7	0.285E+6	4.58	4.53
10.24	38.53	0.0826	0.984E-2	0.138E+7	0.137E+7	0.296E+6	4.68	4.63
10.24	36.70	0.1015	0.206E-1	0.127E+7	0.125E+7	0.285E+6	4.47	4.38
10.95	37.22	0.0922	0.134E-1	0.146E+7	0.144E+7	0.322E+6	4.55	4.48
11.02	38.67	0.0839	0.816E-2	0.154E+7	0.152E+7	0.339E+6	4.53	4.49
11.02	37.07	0.0990	0.202E-1	0.144E+7	0.141E+7	0.326E+6	4.41	4.32
11.62	36.29	0.1032	0.154E-1	0.151E+7	0.149E+7	0.352E+6	4.28	4.22
11.81	38.27	0.0860	0.957E-2	0.168E+7	0.166E+7	0.379E+6	4.43	4.38
12.13	36.22	0.1111	0.220E-1	0.158E+7	0.155E+7	0.382E+6	4.15	4.06

Table 1. Forty-five subpopulations of size 10,000 ordered by increasing metric log diameter — con.

Metric volume weighted average			Scribner defect	Total gross Scribner	Total net Scribner	Total net metric	BFDCF	BFDCF
diameter[1]	length[2]	taper[3]	fraction[4]	volume[5]	volume[6]	volume[7]	GN[8]	NN[9]
12.60	37.50	0.0919	0.103E-1	0.177E+7	0.175E+7	0.417E+6	4.25	4.20
12.60	37.78	0.0904	0.183E-1	0.181E+7	0.178E+7	0.423E+6	4.29	4.21
12.75	36.16	0.1149	0.225E-1	0.174E+7	0.170E+7	0.418E+6	4.15	4.05
13.39	37.09	0.0990	0.114E-1	0.212E+7	0.209E+7	0.463E+6	4.57	4.51
13.39	37.54	0.0972	0.170E-1	0.217E+7	0.213E+7	0.470E+6	4.62	4.54
14.04	36.10	0.1118	0.194E-1	0.225E+7	0.220E+7	0.492E+6	4.57	4.48
14.17	38.15	0.0980	0.163E-1	0.250E+7	0.246E+7	0.533E+6	4.68	4.60
14.35	36.80	0.1097	0.256E-1	0.244E+7	0.238E+7	0.520E+6	4.70	4.58
14.96	37.04	0.1017	0.188E-1	0.272E+7	0.267E+7	0.565E+6	4.81	4.72
15.29	36.83	0.1109	0.263E-1	0.286E+7	0.279E+7	0.590E+6	4.85	4.72
15.75	37.42	0.1025	0.159E-1	0.318E+7	0.313E+7	0.629E+6	5.05	4.97
15.89	37.30	0.1079	0.239E-1	0.324E+7	0.317E+7	0.641E+6	5.06	4.94
16.53	37.39	0.1032	0.180E-1	0.355E+7	0.349E+7	0.687E+6	5.17	5.08
17.18	37.10	0.1116	0.181E-1	0.383E+7	0.376E+7	0.734E+6	5.21	5.12
17.63	37.21	0.1119	0.229E-1	0.413E+7	0.403E+7	0.776E+6	5.32	5.20
18.57	37.26	0.1116	0.210E-1	0.463E+7	0.453E+7	0.850E+6	5.45	5.33
19.41	37.22	0.1155	0.254E-1	0.512E+7	0.499E+7	0.927E+6	5.53	5.39
20.76	36.96	0.1174	0.282E-1	0.621E+7	0.603E+7	0.104E+7	5.98	5.81
22.82	36.58	0.1218	0.322E-1	0.744E+7	0.720E+7	0.122E+7	6.10	5.90
29.94	34.78	0.1375	0.530E-1	0.120E+8	0.114E+8	0.181E+7	6.64	6.29

[1]Small-end metric diameter in inches, recorded as radius in centimeters and then converted to diameter in inches, weighted by metric volume.

[2]Metric length in feet, converted from decimeters, weighted by metric volume.

[3]Metric taper: ((large-end radius in cm - small-end radius in cm) x 2)/(2.54 x metric length in feet), weighted by metric volume.

[4](gross Scribner volume - net Scribner volume)/(gross Scribner volume), weighted by gross Scribner volume.

[5]Total gross Scribner volume in board feet.

[6]Total net Scribner volume in board feet.

[7]Total net metric volume in cubic feet. This volume is calculated using firmwood-based rounding and defect deduction conventions.

[8]The observed gross Scribner board foot to net cubic foot ratio. The net cubic foot value is calculated using firmwood-based rounding and defect deduction conventions.

[9]The observed net Scribner board foot to net cubic foot ratio. The net cubic foot value is calculated using firmwood-based rounding and defect deduction conventions.

Table 2. Twenty-two subpopulations of size 20,000 ordered by increasing metric log diameter[1]

Metric volume weighted average			Scribner defect	Total gross Scribner	Total net Scribner	Total net metric	BFDCF	BFDCF
diameter	length	taper	fraction	volume	volume	volume	GN	NN
6.30	32.99	0.1171	0.175E-1	0.860E+6	0.845E+6	0.234E+6	3.67	3.61
6.97	38.14	0.0890	0.475E-2	0.121E+7	0.121E+7	0.314E+6	3.86	3.84
7.40	35.54	0.1082	0.100E-1	0.114E+7	0.113E+7	0.323E+6	3.52	3.48
7.92	36.57	0.1031	0.125E-1	0.134E+7	0.133E+7	0.371E+6	3.62	3.57
8.66	37.71	0.0833	0.105E-1	0.160E+7	0.158E+7	0.430E+6	3.72	3.68
9.25	36.23	0.0962	0.128E-1	0.177E+7	0.175E+7	0.467E+6	3.79	3.74
9.45	38.40	0.0812	0.753E-2	0.211E+7	0.209E+7	0.513E+6	4.10	4.07
9.89	36.24	0.1018	0.178E-1	0.220E+7	0.216E+7	0.527E+6	4.17	4.10
10.24	37.63	0.0918	0.150E-1	0.266E+7	0.262E+7	0.581E+6	4.58	4.51
10.99	37.97	0.0879	0.107E-1	0.300E+7	0.297E+7	0.661E+6	4.54	4.49
11.34	36.67	0.1012	0.177E-1	0.294E+7	0.289E+7	0.678E+6	4.34	4.27
11.97	37.24	0.0986	0.156E-1	0.326E+7	0.321E+7	0.761E+6	4.29	4.22
12.60	37.64	0.0912	0.143E-1	0.358E+7	0.353E+7	0.839E+6	4.27	4.21
13.09	36.65	0.1065	0.164E-1	0.385E+7	0.379E+7	0.882E+6	4.37	4.30
13.72	36.80	0.1047	0.182E-1	0.442E+7	0.434E+7	0.962E+6	4.59	4.51
14.26	37.48	0.1038	0.209E-1	0.494E+7	0.484E+7	0.105E+7	4.69	4.59
15.13	36.93	0.1064	0.227E-1	0.558E+7	0.545E+7	0.115E+7	4.83	4.72
15.82	37.35	0.1052	0.199E-1	0.642E+7	0.630E+7	0.127E+7	5.06	4.96
16.87	37.24	0.1075	0.180E-1	0.738E+7	0.725E+7	0.142E+7	5.19	5.10
18.12	37.23	0.1117	0.219E-1	0.876E+7	0.857E+7	0.163E+7	5.39	5.27
20.12	37.08	0.1165	0.269E-1	0.113E+8	0.110E+8	0.196E+7	5.77	5.61
27.07	35.51	0.1312	0.450E-1	0.194E+8	0.186E+8	0.303E+7	6.42	6.13

[1]See the footnotes to Table 1.

Table 3. Eleven subpopulations of size 40,000 ordered by increasing metric log diameter[1]

Metric volume weighted average			Scribner defect	Total gross Scribner	Total net Scribner	Total net metric	BFDCF	BFDCF
diameter	length	taper	fraction	volume	volume	volume	GN	NN
6.68	35.94	0.1010	0.100E-1	0.207E+7	0.205E+7	0.549E+6	3.78	3.74
7.68	36.09	0.1055	0.114E-1	0.248E+7	0.245E+7	0.694E+6	3.57	3.53
8.97	36.94	0.0900	0.117E-1	0.337E+7	0.333E+7	0.896E+6	3.76	3.71
9.67	37.31	0.0916	0.128E-1	0.430E+7	0.425E+7	0.104E+7	4.14	4.08
10.64	37.81	0.0897	0.127E-1	0.566E+7	0.559E+7	0.124E+7	4.55	4.50
11.67	36.97	0.0998	0.166E-1	0.621E+7	0.610E+7	0.144E+7	4.31	4.24
12.85	37.13	0.0990	0.154E-1	0.743E+7	0.732E+7	0.172E+7	4.32	4.25
14.00	37.16	0.1042	0.196E-1	0.936E+7	0.917E+7	0.202E+7	4.64	4.55
15.49	37.15	0.1058	0.212E-1	0.120E+8	0.117E+8	0.242E+7	4.95	4.85
17.54	37.24	0.1098	0.201E-1	0.161E+8	0.158E+8	0.305E+7	5.30	5.19
24.34	36.13	0.1254	0.383E-1	0.308E+8	0.296E+8	0.499E+7	6.16	5.93

Table 4. Five subpopulations of size 90,000 ordered by increasing metric log diameter[1]

Metric volume weighted average			Scribner defect	Total gross Scribner	Total net Scribner	Total net metric	BFDCF	BFDCF
diameter	length	taper	fraction	volume	volume	volume	GN	NN
7.15	36.26	0.1018	0.103E-1	0.514E+7	0.508E+7	0.138E+7	3.72	3.69
9.47	37.32	0.0897	0.119E-1	0.906E+7	0.895E+7	0.223E+7	4.06	4.01
11.65	37.33	0.0953	0.152E-1	0.141E+8	0.138E+8	0.322E+7	4.36	4.30
14.34	37.04	0.1050	0.193E-1	0.220E+8	0.215E+8	0.468E+7	4.69	4.60
21.33	36.60	0.1186	0.315E-1	0.501E+8	0.486E+8	0.868E+7	5.78	5.60

Table 5. All 455,382 logs[1]

Metric volume weighted average			Scribner defect	Total gross Scribner	Total net Scribner	Total net metric	BFDCF	BFDCF
diameter	length	taper	fraction	volume	volume	volume	GN	NN
15.85	37.17	0.1073	0.237E-1	0.101E+9	0.983E+8	0.203E+8	4.97	4.85

[1]See the footnotes to Table 1.

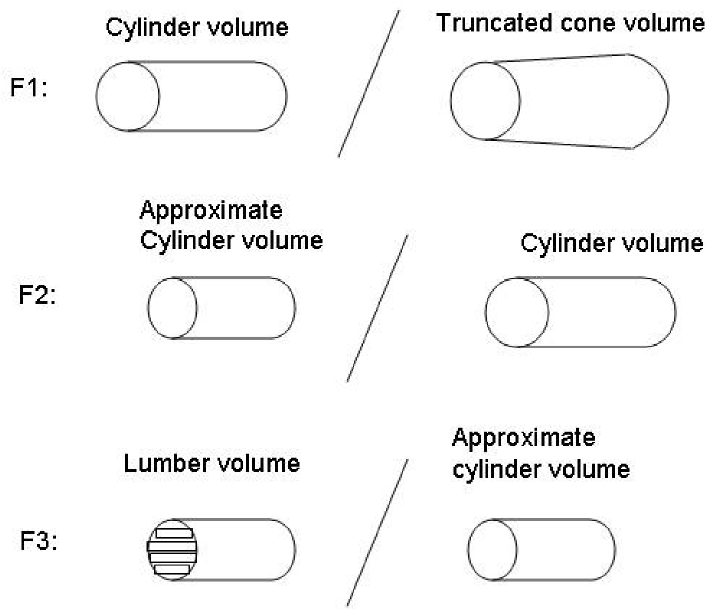

Figure 1 – The F_1, F_2, and F_3 factors.

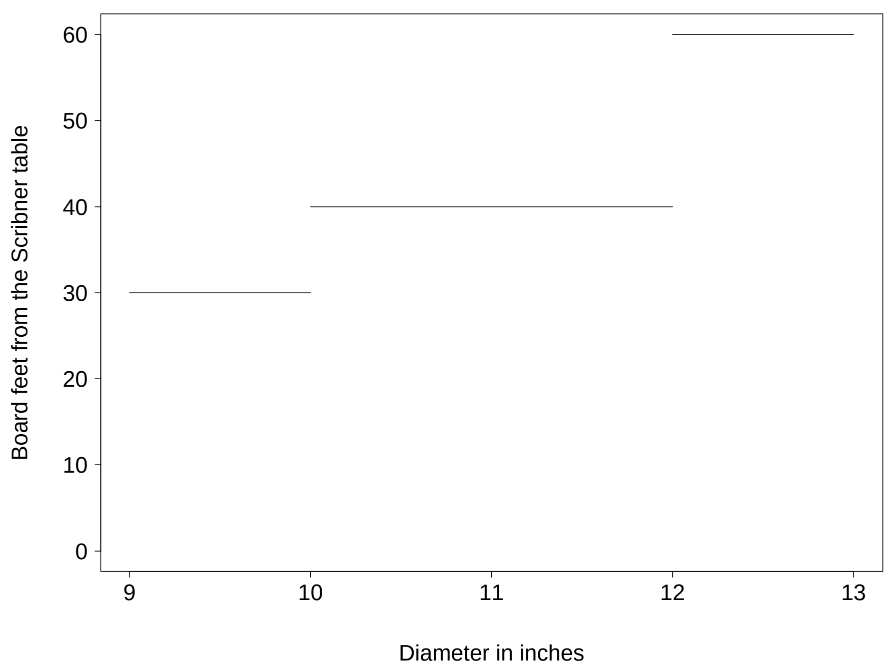

Figure 2 – Board feet from the Scribner table for a length fixed at 12 feet (assumes West Coast truncation rules).

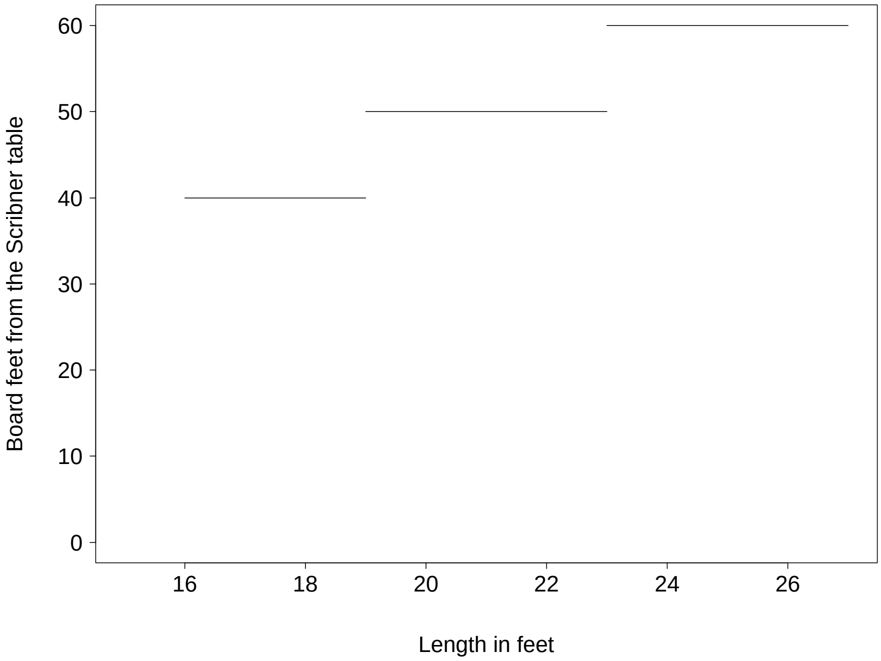

Figure 3 – Board feet from the Scribner table for a diameter fixed at 9 inches (assumes West Coast truncation rules).

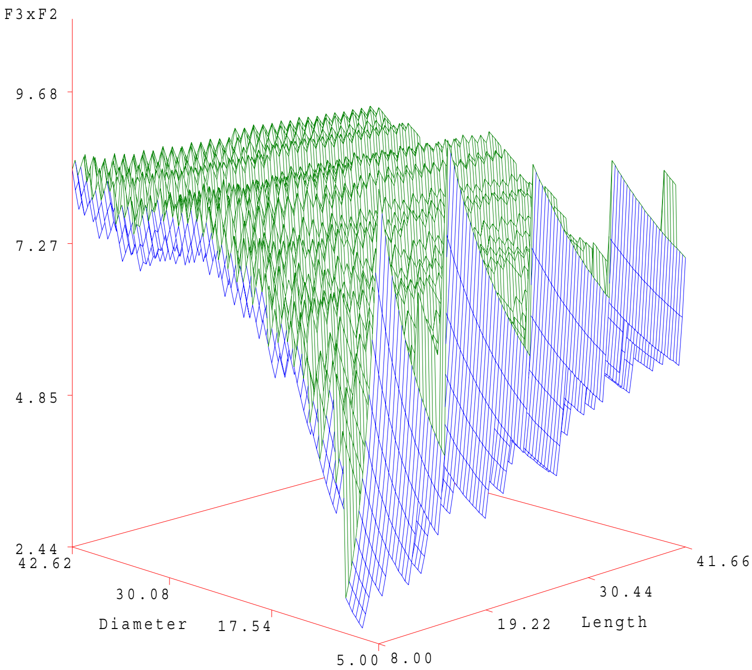

Figure 4 – The unsmoothed $F_3 \times F_2$ surface.

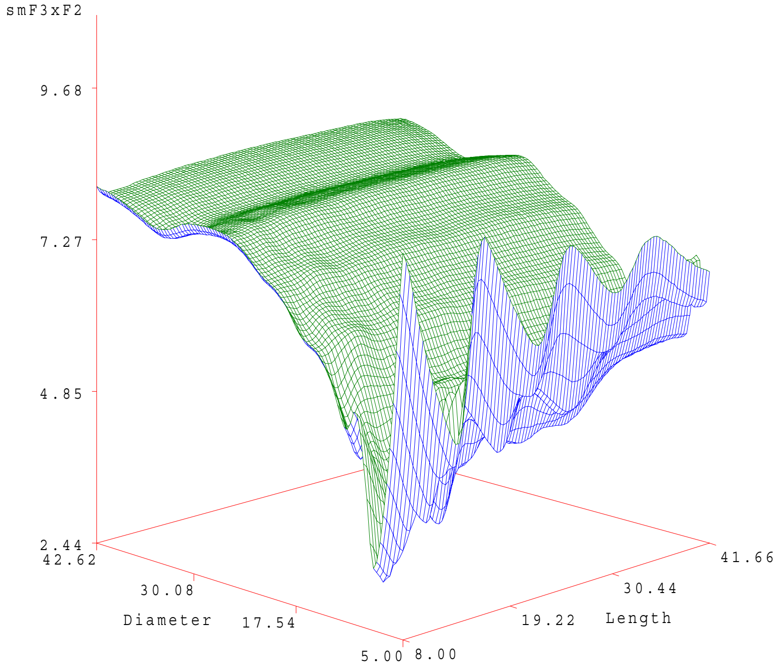

Figure 5 – The smoothed $F_3 \times F_2$ surface with smoothing parameter α set to 0.05.

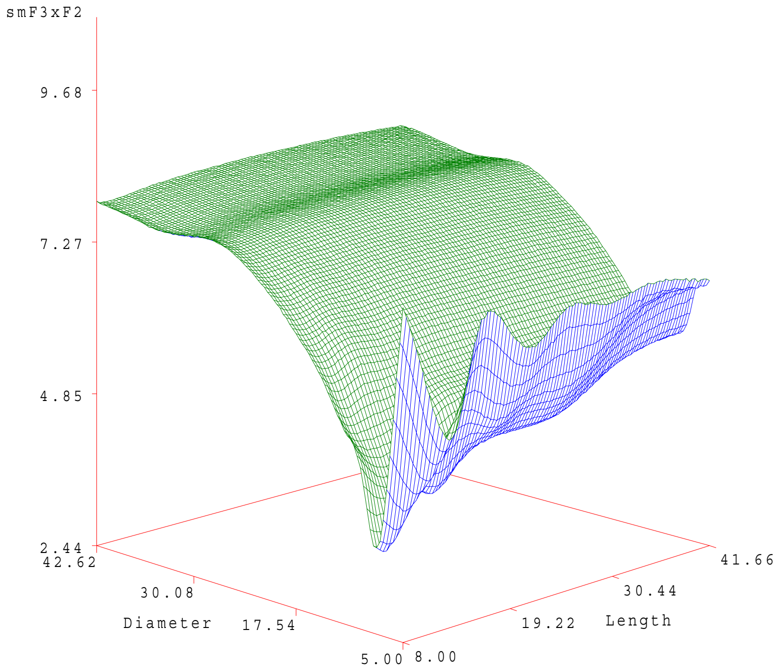

Figure 6 – The smoothed $F_3 \times F_2$ surface with smoothing parameter α set to 0.10.

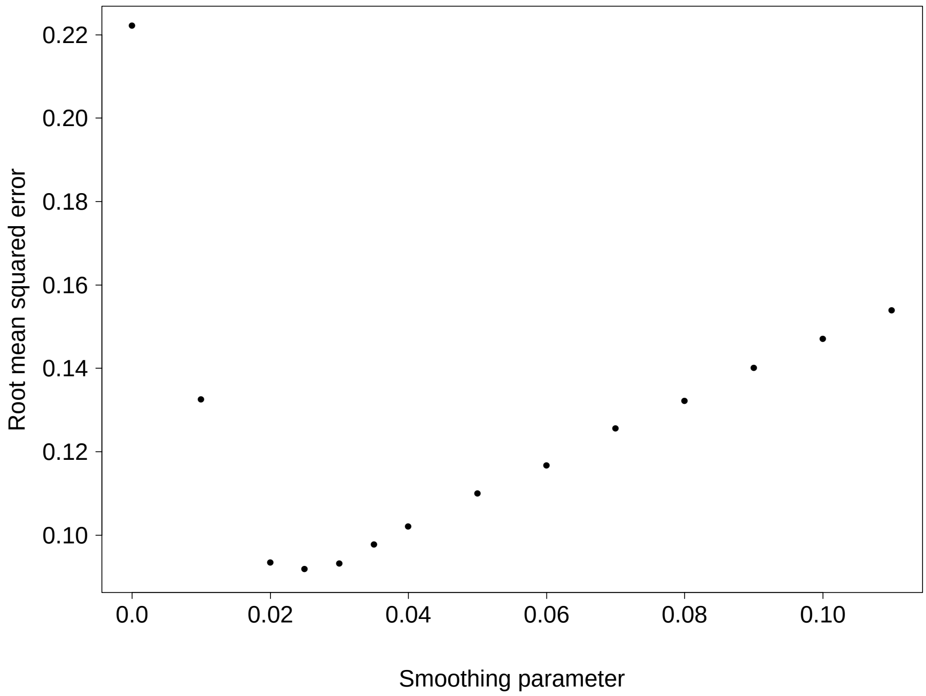

Figure 7 – From the fits of the observed ratios to the predicted ratios, 45 subpopulations.

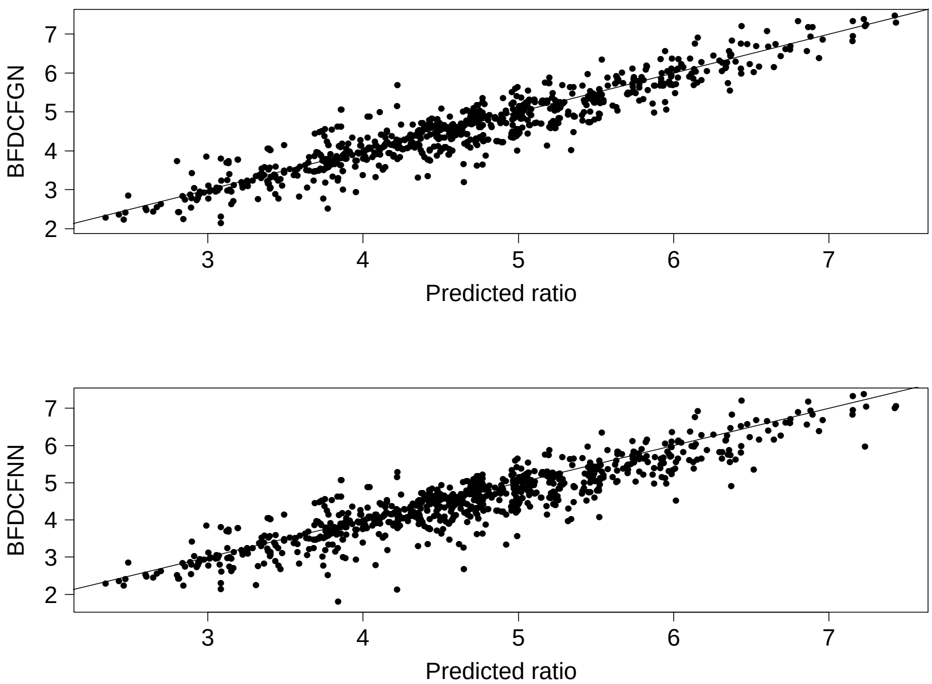

Figure 8 – Observed ratios versus (uncalibrated) predicted ratios ($F_3 \times F_2 \times F_1$).

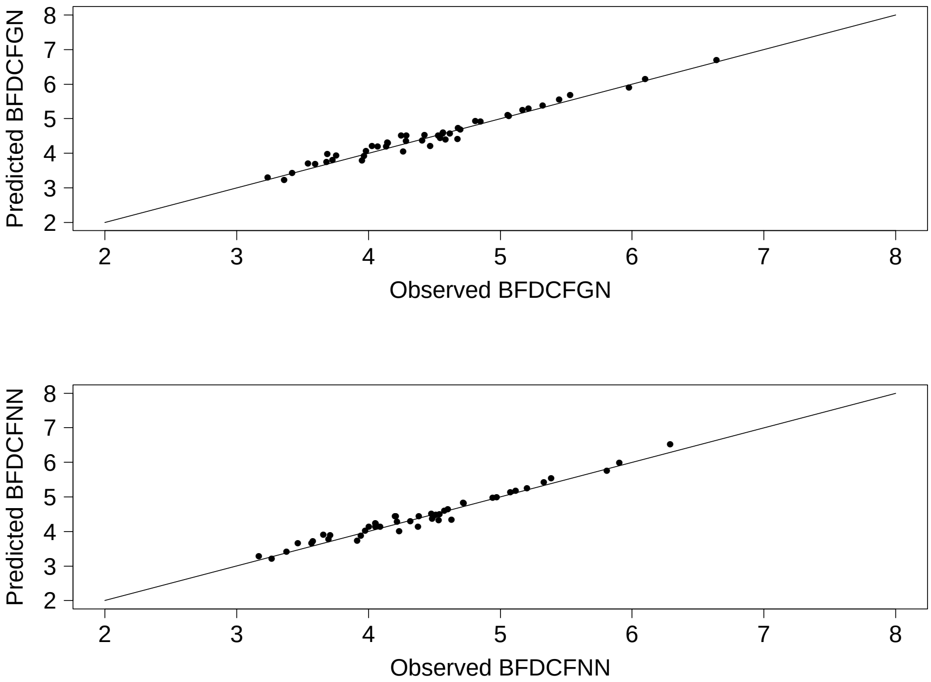

Figure 9 – Calibrated predicted ratios versus observed ratios, 45 subpopulations of size 10,000.

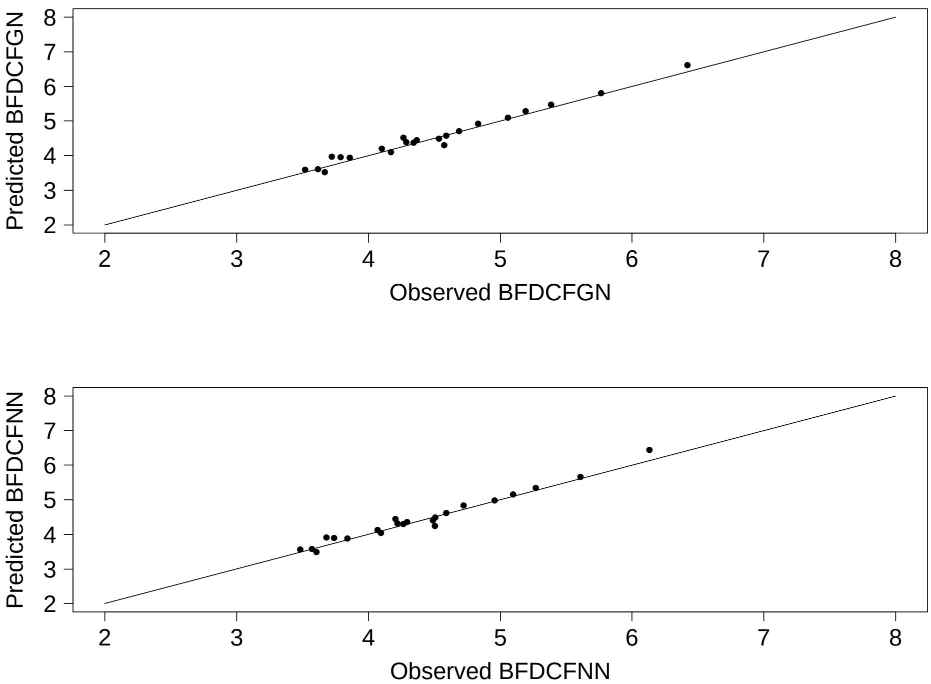

Figure 10 – Calibrated predicted ratios versus observed ratios, 22 subpopulations of size 20,000.

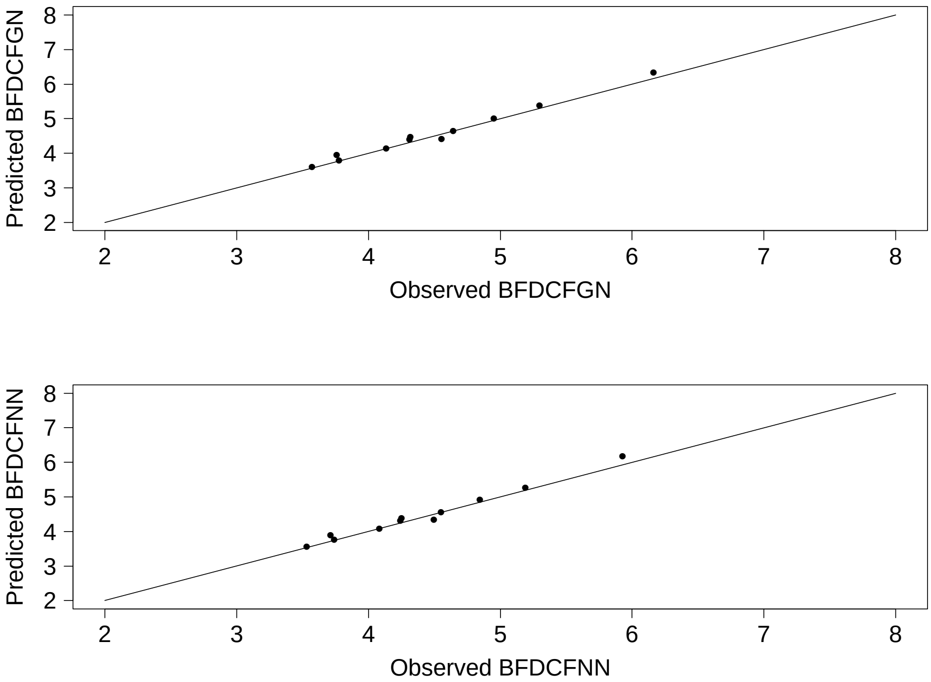

Figure 11 – Calibrated predicted ratios versus observed ratios, 11 subpopulations of size 40,000.

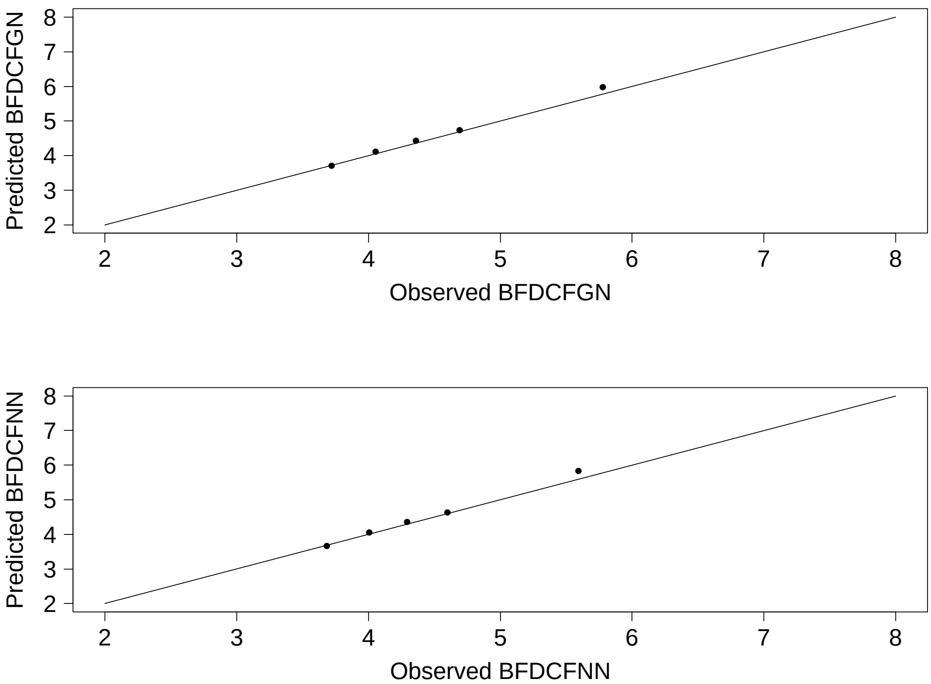

Figure 12 – Calibrated predicted ratios versus observed ratios, 5 subpopulations of size 90,000.